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THEORY OF CONSTRAINTS FOR DESIGN AND MANUFACTURE OF THERMOPLASTIC PARTS

David Kazmer, University of Massachusetts Amherst
Christoph Roser, University of Massachusetts Amherst
Stephen Shuler, GE Plastics

Abstract

The functionality and manufacturability of a given product is largely determined by constraints imposed by the end user and manufacturer. With conflicting constraints, the designer may be unable to satisfy the complete set and has to find the best trade off to optimize the design. In order to find this point with the minimum quality loss, extensive knowledge of the design space is essential but seldom readily available. The design space can be investigated using various Design of Experiments techniques, such as a Taguchi L128 experimental design with augmented levels [1]. The actual loss of performance or cost can be calculated using a loss function for each constraint and then combined in order to find the total quality loss and subsequently the optimal point in the design space. Application of the developed theory will then be demonstrated.

Introduction

Injection molding is a commonly used process for the manufacture of commercial goods. The design of molded parts, however, poses some significant challenges to the product designer as well as the manufacturing engineer. In particular, there is significant coupling between the product geometry, material properties, and process dynamics. Selection of wall thickness, for instance, is dependent upon the required part stiffness, elastic modulus of the polymer, and gating design with its implications on pressure drop and flow length.

The trade-offs between the many design and process requirements can often be made more easily by increasing the part cost. In the above example, for instance, the requirements can be more easily met by increasing the wall thickness, increasing the number of gates, or using a more rigid material. However, each of these decisions will typically increase the part cost.

What makes a design competitive? Largely the ability of the development team to adeptly manage the trade-offs between attributes which are critical to quality. This paper explores these concepts in more detail.

A Theory of Constraints

From a fundamental viewpoint, the value of a product is determined by the functionality that it provides the customer. Philosophically, this functionality is related to the requirements and constraints placed on the product. At the beginning of the product development, no requirements have been set. While the design space is totally unconstrained, vague product concepts provide no value. As development continues, both the requirements and the design become more defined. Each additional product requirement typically adds some value to the customer. Each additional product requirement, however, provides a constraint on the design, which is another possibility for the product design to become unacceptable.

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This paper seeks to answer the question: How can the product and process design parameters be selected to ensure that all product requirements are met? A theory of constraints has been developed elsewhere to provide a rational basis for performance evaluation [2]. This work states that 1) feasible designs must satisfy all imposed constraints, and 2) the optimal design will occur at a position such that the probability of violating any constraint is minimized.

Consider a vector composed of product and process design variables, X. Each of the product and process variables can have stochastic behavior, i.e. distributed according to some probability density function f(X). Defining y to be a single response g(X), the estimated value of y can be defined as:

$$E(y(X)) = \int_{-\infty}^{\infty} g(X) f(X) dX \quad (1)$$

Given this estimate of y, we must now evaluate its value relative to the product requirements. Several different approaches have been advocated. Taguchi has defined a quality loss function as the response deviates from target [3]. A classic optimization approach might define an objective function which represents the value of the product performance [4]. Utility theory might utilize customer lottery questions to explicitly evaluate the utility of choosing between different performance options. [5]

Each of these approaches, however, is deficient. Taguchi’s approach does not stem from a fundamental basis and provides very low fidelity regarding the product performance. Utility theory and classic optimization fail on two fronts. First, these approaches are not well suited to probabilistic design and process variables. Second, determination of a single aggregate measure to determine the product value is extremely difficult.

As such, this approach has defined the value or utility, u, of the product response as the probability that the response meets the product specification. For instance, consider a requirement that y must be less than an upper specification limit, USL. The utility is then defined by:

$$\begin{aligned} u(x) &= P(y \le USL) \\ u(x) &= \int_{-\infty}^{USL} f_y(y) dy \end{aligned} \quad (2)$$

This approach is notable in that it stems from a completely rational basis. As such, this measure of utility is directly understandable as the likelihood of product acceptance, and is operable as such. All products, however, have multiple product and process specifications. Injection molded parts typically have many dozens of specifications: product requirements such as weight, size, stiffness, strength, aesthetics coupled with process requirements such as cycle time, moldability, and warpage.

Each of these specifications will have a utility associated with it. Remembering that the utility represents the likelihood of acceptance, a single measure of global utility, R, can be estimated by multiplying all the the utilities together:

$$\Re = \prod_{i=1:n} u_i \quad (3)$$

By this definition, R is the joint probability that the product will meet all of the specifications. A higher R represents a more robust design where the limit of 100% indicates that the product will be acceptable given the defined product and

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process noise. If a constraint is violated, then the likelihood of the specification being met approaches 0 and the global measure of utility will likewise approach 0.

This global norm is founded on a rational basis and is also directly operable. For instance, $\Re$ can be related to the process capability index, Cp, through the inverse normal cumulative probability density function, F^{-1}:

$$Cp = \frac{F^{-1}\left(\frac{1-\Re}{2}\right)}{3} \qquad (4)$$

Implementation

The described theory of constraints provides a rational basis for establishing the ‘feasible window’ for product and process design. Given the approach’s ability to consider stochastic behavior of product and process variables, the same theory of constraints also provides a method for assessing product quality. The primary drawback of the method, however, is its reliance on developing a global model for each performance characteristic. This implementation will be described in the next two subsections.

Response Surface Methodology

The calculation of design utility, eq. 2, requires the estimation of each response’s probability density function, fy. As a response, this is an extremely difficult function to obtain from experimental data, requiring multiple samples at many points through the design universe to characterize the nature of the performance distribution. Fortunately, the probability density function of a response can be related to the probability density function of its independent variables [6]:

$$f_y(y) = \frac{f_x(x)}{\left|g'(x)\right|}. \qquad (5)$$

Calculation of the derivative of the response, g’(x), is not trivial, however. One approach to estimation of g(x) is the use of response surface modeling [7] through Design of Experiments (DOE). The current implementation is able to use two different Design of Experiments to model the response surfaces, the Box Behnken Method and the Central Composite Design (CCD). Both designs incorporate three levels and are therefore able to model response surfaces including constant, first order, linear interactions, and quadratic effects. Throughout this paper, these three levels are denoted as –1, 0, +1 for the lower, middle and upper level of each design and process variable.

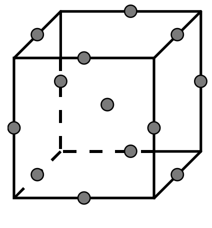

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Figure 1: Box Behnken Design

The Box Behnken Design investigates every possible combination of two factors out of all factors, with those two factors set to the extreme limits while the other factors are set to zero. After investigating every combination, a center point run is added. This experimental design is useful to estimate linear effects, quadratic effects and all linear 2-way interactions. However, because no corner points are used it is not possible to incorporate factors with only two possible settings, like e.g. two types of material. In addition, for more than four factors the number of runs required is greater than the number of runs for a CCD.

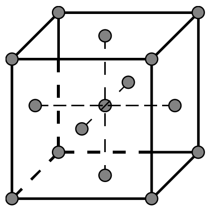

Figure 2: Central Composite Design

The CCD uses a different approach to model the response surface. This experimental design consists of three parts. First, the CCD utilizes a normal full factorial design. Then, these runs are augmented with a center point run. Finally, the quadratic effects are modeled by making two runs for each factor, where the factor is set to $+\alpha$ and $-\alpha$ while all other factors are zero. The value for α is recommended to be $n_f^{1/4}$, where n_f is the number of runs in the factorial part of the design.

Unfortunately, this valuation of α could require experimentation at extreme settings for a large number of factors. For example, ten factors would require 1024 factorial runs and a value of α equal to 5.65. For a melt temperature with a normal range from 200 to 240 ºC (380 to 460 F) as ±1 would have to be run at 127 and 331 ºC (260 and 595 F) for $\alpha=\pm$ 5.65. These melt temperatures are way outside any useful range and the experimentation would not prove feasible reducing the characterization of the response models. This problem can be overcome by setting $\alpha=\pm1$. This greatly lowers the prediction of the variance of the results, but the variance is of no concern for the response surface modeling since the variances will be estimated through (5). A representative CCD design for three factors is shown in Table 1.

Table 1: Central Composite Design

b0	A	B	C	AB	AC	BC	ABC	A²	B²	C²
1	-	-	-	+	+	+	-	1	1	1
1	-	-	+	+	-	-	+	1	1	1
1	-	+	-	-	+	-	+	1	1	1
1	-	+	+	-	-	+	-	1	1	1
1	+	-	-	-	-	+	+	1	1	1
1	+	-	+	-	+	-	-	1	1	1
1	+	+	-	+	-	-	-	1	1	1
1	+	+	+	+	+	+	+	1	1	1
1	0	0	0	0	0	0	0	0	0	0
1	+	0	0	0	0	0	0	+	0	0
1	-	0	0	0	0	0	0	+	0	0
1	0	+	0	0	0	0	0	0	+	0
1	0	-	0	0	0	0	0	0	+	0
1	0	0	+	0	0	0	0	0	0	+
1	0	0	-	0	0	0	0	0	0	+

The first column is the constant coefficient, followed by the main effects, all interactions and finally the quadratic effects. This provides a system of equations, with:

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$$X*B=Y, \qquad (6)$$

where X is the design matrix and B is a column vector with the model parameters for each column of X. Y is a matrix with the experimental results for each investigated quality specification in the columns and the related results in the corresponding rows. Transforming the matrices in the following way solves this system of equations:

$$B= (X^T*X)-1* X^T *Y \qquad (7)$$

Thus B provides the coefficients for the prediction equation. The result Y is obtained by multiplying a row vector X, incorporating the settings, the interaction values, and their quadratic values with the vector B.

Performance Data

While the theory surrounding response surface modeling is fairly involved, its implementation can be embedded in software and completely removed from the design or process engineer. The described response surface modeling provides a method for efficiently sampling the design space and characterizing the product response.

The product performance data can originate from either simulation runs or molding trials depending on the current state of product development. The authors envision a 'Design for Six Sigma' strategy utilizing these same concepts throughout the product development cycle. For instance, application engineers could use this methodology with finite element analysis to ensure robust geometric designs. Then mold-filling analyses could be utilized to ensure moldability across a wide range of process conditions. Finally, the same tool can be used by the processing engineer to set-up and optimize the injection molding process.

Once performance data is supplied, the engineer can evaluate the global product performance. The effect of changing product specifications or input variable distributions can be immediately witnessed. This information is critical is assessing the engineering feasibility and economic viability during product development.

Software

The described theory has been implemented in a software program named Looking Glass to provide the industry practitioner a dynamic view of the injection molding (IM) process. The user will be able to determine critical process parameter and find a process window for their application and molding process. Graphics visualize the process characteristics throughout the program to improve the understanding of the IM process for the user.

Using this software, the user can select the critical to quality specifications (CTQs). For each product requirement, the user must specify lower and/or upper specification limits. Depending on the product requirements, the user chooses up to eleven parameters for investigation and gives a range, e.g. melt temperatures from 160 to 260 C and fiber contents from 5 to 20 %. It is interesting to note that the selection of independent variables will vary with the background of the user and stage of product development. For instance, product designers may be more interested in wall thickness, elastic modulus, and number of ribs while a process engineering may be more interested in melt temperature, injection speed, and pack pressures. However, all design and process variables are available for investigation.

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Next the program creates a DOE including quadratic effects as previously discussed. Finally, the performance data is provided through simulation or experimentation. according to the settings provided by the DOE. The software automatically solves the prediction equations and provides many different types of results to the user.

Results and Discussion

The previously described theory will now be demonstrated through a simple molding application. Consider a flat plaque to be molded of ABS with approximate dimensions of 100 mm by 20 mm by 2 mm. A typical commercial application may be concerned about the trade-off between cooling time, material utilization, and stiffness.

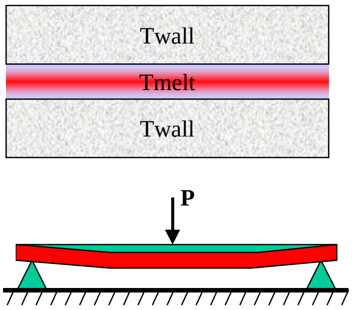

Figure 3: Geometries for cooling and deflection analysis

Given the variable declarations as defined in the Nomenclature, the solution to these problems can be roughly estimated as:

$$T(z,t) = T_{wall} + \left(T_{melt} - T_{wall}\right) \cdot erf\left(\frac{z}{\sqrt{2\alpha t}}\right) \tag{8}$$

$$\delta(h,E) = \frac{PL^3}{4Ewh^3} \tag{9}$$

The question we want to address is: "what are the design and process parameters that will result in minimal cooling time and deflection?" This question is made significantly more difficult given that there may be issues surrounding the exact mold temperature, melt temperature, elastic modulus, and load that the plaque will experience. Given an input distribution, the utility of cooling time as a function of wall thickness can be plotted (Figure 4).

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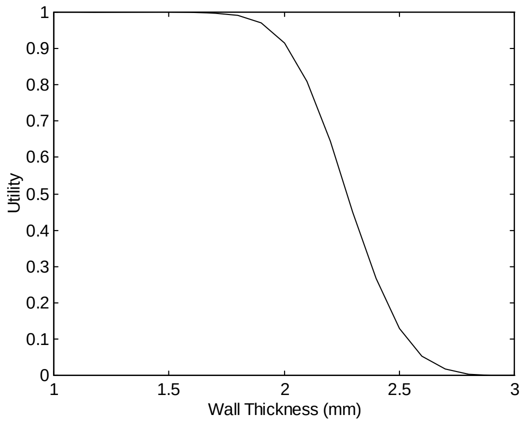

Figure 4: Utility of Cooling Time vs. Thickness

The curve indicates that wall thicknesses less than 2 mm will result in an adequately small cooling time of less than 10 sec. At higher wall thicknesses, machine settings, material properties, and process dynamics will reduce the probability that the cooling time will be acceptable. It is important to realize that this is one plot of utility for a constant melt temperature and Young's Modulus. Changes in the input distributions will change the utility curves, which can fortunately be modeled through the use of equations 1 and 2.

The one utility curve for cooling time as a function of wall thickness does not provide sufficient information to make an informed design decision. In fact, the relationships between cooling time and deflection are quite involved – yet some trade-off has to be made between cooling time and deflection. The global metric of utility, $\Re$, can be calculated as shown in equation 3 as a function of all the performance specifications. The resulting plot of global utility is shown in Figure 5. In the figure, utility is indicated on the vertical axis as a function of melt temperature and wall thickness. The results indicate the feasible region in which cooling time and deflection are acceptable. If the design was not feasible, then the utility across the entire design and process space would approach zero.

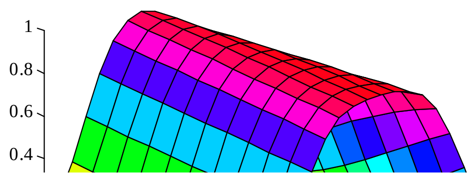

Figure 5: Utility vs. Thickness and Temperature

The figure indicates that a wall thickness close to 2 mm is optimal with a melt temperature at 230 C. The parabolic shape is due to the conflicting objectives placed on the design. At thicker wall sections, the cooling time is unacceptable and

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the utility drops off. At thinner wall sections, the deflection becomes significant and the design is unacceptable. The surface does show an interaction: at lower melt temperatures a thicker wall section does have higher utility since the cooling time is reduced. Similar graphs of utility can be drawn for other design variables such as elastic modulus, mold temperature, etc. In this way, the global measure of utility can be used for many product and process design decisions.

The plaque example in this paper was provided to demonstrate the theory of constraints and associated utility functions. The resulting design solution could likely be improved by changing the design configuration. As such, this design approach is being applied to the development of more complex commercial applications including both parametric changes (melt temperature, mold temperature, thickness) and alternative design concepts (topology, number of gates, number of ribs).

Finally, it should be noted that the estimate of utility, R, stems from a rational basis and can be interpreted directly as the likelihood of success. As such, the metric can be related to other common measures of quality such as the process capability, Cp, and Six Sigma principles through use of the metric described in equation 4. Looking Glass then become a powerful tool for not only predicting product performance, but also providing an estimate of the quality levels related to that product performance.

Conclusions

Design of injection molded parts can be a difficult task due to the number of specifications and their complex relationships to product and process design variables. The goal of this paper was to develop a measure of design optimality and relate that measure to the constraints placed on a product. The resulting methods can be used to satisfy multiple objectives in product and process design, even subject to stochastic variation and parameter uncertainty.

Nomenclature

α	thermal diffusivity
B	matrix of coefficients relating X to Y
Cp	process capability index
E	elastic modulus
f	probability density function
F^{-1}	inverse normal cumulative density function
g	performance function relating X to Y
h	part thickness
L	part length
P	load
$\Re$	global measure of design utility
t	time
Tmelt	melt temperature
Twall	mold wall temperature
u	single measure of design utility

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USL	upper specification limit
w	part width
X	product and process design variables
Y	product requirements
z	thickness coordinate

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Key Words

robust design; molded part design; quality issues; design of experiments