Preprint of: Roser, Christoph, and David O Kazmer. “A Method for Robust Flexible Design.” In *Proceedings of the Annual Technical Meeting of the Society of Plastic Engineers*, 3:2948–52. Orlando, Florida, USA: Society of Plastics Engineers, 2000.

A METHOD FOR ROBUST FLEXIBLE DESIGN

Christoph Roser, University of Massachusetts Amherst

David Kazmer, University of Massachusetts Amherst

Abstract

The robust design method seeks to minimize the sensitivity of performance to uncontrolled variation. Product development frequently uses numerical simulations, analytic models and experimental data. However, these underlying predictions may be inaccurate, and include errors that cause the design to be unacceptable and require a design change. This paper presents a method that analyzes the possibility of a design change based on prediction uncertainty; and then estimates possible changes and evaluates the product design flexibility. The results indicate that small changes in design variables may reduce the likelihood and cost of future design changes.

Introduction

The standard engineering approach utilizes the robust design providing the optimum performance within the allowed design space with respect to noise variation. However, this robust point was determined using models, simulations or experiments, and there is a possibility that the physical embodiment of the design might not satisfy the specifications due to possible prediction inaccuracies. Although the design prediction considers noise, it usually does not account for uncertainties and inaccuracies in the predictions of the design performance.

As shown in Figure 1, the robust design method predicts a feasible design window, from which a design is selected according to the utility. However, due to prediction uncertainty, the actual design window might be of different shape, size and location. Due to this lack of consideration for prediction inaccuracies, the finely tuned robust design might violate specifications because the underlying predictions lack the necessary accuracy. If the selected design lies outside of the actual design window, a design change is necessary even if the model predicted this design to be optimal (Roser and Kazmer 1999). This design change will incur unforeseen development costs, and may also alternate the performance and unit cost of the product.

The presented method analyzes the probability of change, possible design changes and the related change cost and expected performance of the improved design. Based on this information, an estimated cost of the design is evaluated to provide a basis for rational comparison of different designs. (Thornton 1999) developed a related method to analyze uncertainty regarding the process capability.

System Description

The following knowledge regarding the design system is required to perform the proposed analysis. The relations between the input variables X and the specified output responses Y has to be known. This relation can be based on experimental data, models or simulations. To reduce computation time it is advisable to consider only the input variables in X, that show a significant effect towards the performance parameter Y (Roser and Kazmer 1998).

Preprint of: Roser, Christoph, and David O Kazmer. “A Method for Robust Flexible Design.” In *Proceedings of the Annual Technical Meeting of the Society of Plastic Engineers*, 3:2948–52. Orlando, Florida, USA: Society of Plastics Engineers, 2000.

This functional relation assumes a deterministic case. Due to noise, however, the design responses will be of probabilistic nature. Therefore, the noise occurring in the design variables X and the design responses Y have to be estimated. Within this method, noise is described as a probability density function $pdf_N(y_i)$, which could be e.g. a normal distribution. There are different ways to predict the response distribution based on the various distributions. A functional evaluation (Papoulis 1991) is usually avoided due to the significant analytic effort required. Standard distributions are frequently assumed to simplify the computation process (Devore 1995). Monte Carlo methods are also commonly used (Suresh 1997). Based on this information, the probability of specification satisfaction P^N_j is estimated based on the noise distributions of the design responses $pdf_N(y_i)$:

$$P_j^N = P\left(LSL_j \leq y_j \leq USL_j\right) = \int_{LSL_j}^{USL_j} pdf_N\left(y_j\right) dy_j \qquad [1]$$

where LSL_j and USL_j represent the lower and upper specification limits. Based on the probability of specification satisfaction P^N_j, the joint probability of multiattribute satisfaction due to noise P^N, i.e. the yield, is estimated by multiplying the individual probabilities as shown below.

$$P^N = \prod_{j=1}^{m} P_j^N \qquad [2]$$

In order to compare different designs quantitatively, the yield is furthermore combined with the estimated marginal cost into a cost utility C^U.

As mentioned in the introduction, the simulations and models used to predict the design responses are not necessarily accurate but may include some prediction uncertainties. These prediction uncertainties can be modeled as probability distributions $pdf_U(y_i)$ similar to noise, and subsequently a combined response distribution can be evaluated. The combined response distribution $pdf_C(y_i)$ can be predicted using similar mathematical procedures as for the evaluation of the noise distribution mentioned above. In addition, the probability of specification satisfaction for the combined distribution P^C_j can also provide an estimate of the joint probability of specification satisfaction in a similar way.

Design Flexibility Analysis

An overview of the method to estimate the expected cost of the design including possible design changes is shown in Figure 2. The methodology analyzes all possible design change options for a given initial design D^S. After determining all possible variable combinations, the design is optimized to increase the combined yield P^C by changing only selected variables. This optimization is performed for all variable combinations within the design space. The cost of the design change is also estimated. This list of possible design changes is then reduced to eliminate redundant changes and undesired changes. Based on this reduced list, the expected design cost is integrated and the probability of project failure is estimated. This result can then be used to enable a flexible design that minimizes cost while leaving adequate adjustments for error.

Design Change Variable Combinations

In order to determine a flexible design, possible combinations of changes in the design variables have to be analyzed and the cost and impact on the design performance compared. If there exist m investigated input variables, there will be 2^m possible sets of design change combinations, representing 2^m subsets of the m dimensional design space. Figure 3 visualizes a three-dimensional design space with the one and two-dimensional subspaces, where only one or two variables are changed.

Preprint of: Roser, Christoph, and David O Kazmer. “A Method for Robust Flexible Design.” In *Proceedings of the Annual Technical Meeting of the Society of Plastic Engineers*, 3:2948–52. Orlando, Florida, USA: Society of Plastics Engineers, 2000.

Optimize Yield Robust against Noise and Uncertainty

Based on the initial design D^S, the design will be optimized with the objective to maximize part quality within the design sub space. Thus, an optimized design D^I_j is created within the design sub space using an objective to maximize the combined yield P^C.

Please note that these design changes are not a list of designs among which the best is to be chosen. Rather, a given design is investigated for the case of excessive defects due to prediction inaccuracies. Also note that these design changes are not suggested changes in case of defects but rather a design flexibility evaluation to determine the trade off between robustness and cost.

This optimization has to be performed for every design sub space with the exclusion of the empty sub space 1, where no design variables are changed and hence the optimized design is identical with the initial design. This set of 2^m optimizations yields a set of changed designs D^I_j, with j ranging from 1 to 2^m. Each of these designs drives a cost utility C^U, a yield due to noise P^N and a combined yield due to noise and uncertainty P^C. These yield and cost estimates will be of importance for the evaluation of the expected cost. Note that some of the optimizations will generate the same design, which is the optimal design for overlapping design subspaces. These duplicate designs have to be eliminated as described below.

Change Cost Analysis

Next, the cost of changing the input variables has to be determined for all possible design parameter combinations. This analysis requires the structuring of the tasks necessary to change a variable. This structure is related to design task modeling, where a design is divided into sub tasks. (Steward 1981) describes the design structure matrix as an approach to manage complex design systems. This approach is extended for the change cost analysis.

The relation between the design variables x_i and the tasks T_k needed to change these design variables are represented in a matrix T. This matrix consists of one row for each design variable x_i, and one column for each possible task T_k. If a change in a design variable x_i requires the execution of task T_k, a 1 will be inserted in the matrix T in row i and column k. Depending on the list of changed design variables, it is possible to determine the tasks required to perform this change. Furthermore, each task T_k is associated with a cost created during the execution of the task. The total cost C_j^C of the design change then evaluates as the sum of the costs of all tasks required changing a set of variables. The total cost C^T of a changed design is then the sum of the cost utility C^U and the change cost C^C.

$$C^T = C^U + C^C \qquad [3]$$

Please note, that the above method for determining the cost of a design change is a very general approach, and estimates the change cost merely based on the changed variables. Improved methodologies for change cost estimations can be developed and used within this methodology.

Eliminate Redundancy

The above design change analysis will yield a list of 2^m designs, optimized for a combined yield P^C in different design sub spaces. However, within this list not all designs are unique. A design might be optimal for different overlapping subspaces and therefore might be listed more than once. In this case, the design with the smaller change cost and time will be selected, and the other instance of the design will be discarded to avoid redundant designs in the list of optimized designs D^I. Not only are redundant designs eliminated, but unfavorable design changes are also removed from the list. Sometimes one

Preprint of: Roser, Christoph, and David O Kazmer. “A Method for Robust Flexible Design.” In *Proceedings of the Annual Technical Meeting of the Society of Plastic Engineers*, 3:2948–52. Orlando, Florida, USA: Society of Plastics Engineers, 2000.
design in the list D^I will be more expensive and less robust than another design in the list D^I. In this case, the more economic and more robust design will always be preferred over the less economic and less robust design as shown below in Figure 4. Therefore, all unfavorable designs will be eliminated from further consideration in the methodology.

The mathematical representation of the described logic is shown below. The set of optimized designs D^I is reduced to a subset D^R. Every design from the set of optimized designs D^I is compared to every other design in the set. If any design has a smaller total cost C^T and a larger combined yield $P^C{}_j$, or there exists an identical design in the remaining list, then the design is not included in the reduced set D^R.

$$\left.\begin{array}{ll} if\left[\left(C_j^T < C_i^T\right) \vee \left(P_j^J > P_i^J\right)\right] \forall j \neq i & \varnothing \\ or\left[\left(C_j^T < C_i^T\right) \vee \left(P_j^J = P_i^J\right)\right] \forall i > j & \\ \quad else & D_j^I \end{array}\right\} \in D^R \forall j \qquad [4]$$

In order to simplify further evaluations, the elements in D^R are sorted according to the total cost C^T. As unfavorable designs are already reduced, this will also cause the elements to be arranged according to the combined yield P^C.

Probability of Design Failure

The above design flexibility analysis created a list of possible feasible design changes D^R. Using this evaluation of the design flexibility, it is possible to estimate the expected cost of the design and the overall probability of design failure in order to enable a trade off between different designs under consideration of the design flexibility as shown below.

Based on the yield due to noise P^N and a combined yield due to uncertainty and noise P^C, it is possible to estimate the yield caused only by prediction uncertainty P^U by dividing the yields as shown below.

$$P^U = \frac{P^C}{P^N} \qquad [5]$$

This represents the probability that the prediction uncertainty does not have negative effects on the design performance. However, even with all variables optimized to generate a robust design, the largest yield due to uncertainty P^U_{max} out of the designs D^R may not reach 100%, i.e. even the most robust design might fail due to prediction uncertainties. In this case, the design space has to be expanded, by conducting a major change in the design concept or by increasing the ranges for some design variables. This probability of failure P^F is evaluated below.

$$P^F = 1 - P^U_{\max} \qquad [6]$$

The consequences of a failed design are difficult to assess a priori. Therefore, the financial impact of a failed design will not be discussed in more detail. However, the information regarding the probability of a failed design is of use to the design team and management to estimate the risk between different design strategies or design projects and aid with the decision where to invest available resources.

Expected Cost

The expected cost of the design D^S can now be evaluated including the effects of design changes by integrating the discrete set of possible design changes D^R. The yield due to uncertainty P^U represents the probability that no design changes are necessary for a given design.

Preprint of: Roser, Christoph, and David O Kazmer. “A Method for Robust Flexible Design.” In *Proceedings of the Annual Technical Meeting of the Society of Plastic Engineers*, 3:2948–52. Orlando, Florida, USA: Society of Plastics Engineers, 2000.

At the design stage, it is not yet known if and how the prediction uncertainties will cause defects in the final design. Hence, it is not yet known if and how the design would have to be changed to adjust for prediction uncertainties. To cope with this uncertainty, the design has to be robust. Therefore, in case of an infeasible design due to prediction uncertainties the design change is assumed to improve the robustness. Depending on the effect of the prediction uncertainty, the robustness has to be adjusted.

The likelihood of change selection can be based on the yield due to uncertainty P^U for the different design change options. The probability of choosing the first design option D^R_1 is identical to its probability of having an accurate prediction, represented by the yield due to uncertainty P^U_1, as shown in Table 1. The second design also has a probability of an accurate prediction P^U_2. However, the more economic design D^R_1 will be preferred if possible. Hence, the probability of selecting the second design D^R_1 is the difference between the yields as shown in Table 1. Similar logic is true for all other designs.

Hence, the estimated cost of the initial design D^S can be integrated by taking the sum of the product of the probability of design selection with the cost of the design as visualized in Figure 5 and evaluated below. Please note that this sum has to be divided by the largest yield due to the remaining probability of project failure P^F.

$$C^E = \frac{C_1^T \cdot P_1^U + \sum_{j=2}^{o} C_j^T \cdot \left(P_j^U - P_{j-1}^U\right)}{P_{Max}^U} \qquad [7]$$

The expected cost of the design was evaluated using the flexibility information. Now it is possible to compare different starting designs using the expected cost C^E including considerations for design changes.

Example: Injection Molded Part

The demonstrated example is a large injection-molded part with a production requirement of 50,000 units. Four significant input variables from geometry, material, and processing parameters may be modified as necessary for the design of this product to deliver adequate response of four constrained quality attributes. These variables are listed in Table 2.

The relations between the design variables and the design responses were modeled as response surfaces using analysis, simulations and experimental data. The probabilistic evaluation was performed assuming normal distributions with error transformations. Three different designs where investigated as listed in Table 3. The first design represent the design optimized for the cost utility C^U, the second the design optimized for the yield due to uncertainty P^U. As thickness is the variable with the largest change cost, the third design has an increased wall thickness *W* compared to the C^U optimal design to reduce the likelihood of a major design change.

Analyzing the third design by evaluating all 16 possible design changes and reducing the redundant changes created a list of possible design changes as shown in Table 4. The cost increases gradually with the yield. The large cost increase for the last two change options is due to the increase in the number of tools used in order to improve the likelihood of satisfying the specification for production time.

A comparison of the first and third design is shown in Figure 6, where the total cost is plotted against the yield due to uncertainty. The integral of the expected cost is also visualized. Note, that the integral for design one is larger than for design three, therefore design one has a larger expected cost than design three. Although design three has a slightly higher total cost

Preprint of: Roser, Christoph, and David O Kazmer. “A Method for Robust Flexible Design.” In *Proceedings of the Annual Technical Meeting of the Society of Plastic Engineers*, 3:2948–52. Orlando, Florida, USA: Society of Plastics Engineers, 2000.
than design one, it is possible to have a greater improvement of the yield P^U by changing only design variables with a small change cost and therefore with only a small cost increase due to the change.

Integrating this flexibility information generates an expected cost of $9.35 for design one and $9.27 for design three. Hence, although design one has a lower cost utility, it is more likely to change and the changes are more expensive than design three. Therefore, design three, although being more expensive according to the cost utility is easier to change and subsequently has a lower expected cost. Hence, design three has a better trade off between the total cost and the design flexibility than design one, and is therefore preferred over design one.

Summary

The described design flexibility analysis investigates possible design changes and determines the expected cost of the design. Based on the design system, the optimal robust design might not necessarily be the best design, as prediction uncertainties may cause design changes and therefore increase the cost of the design. Please note that the probability of failure does not impact this trade off, as this probability is optimal within the design space and therefore identical for every selected design within the design space. The effect of the design changes due to prediction uncertainty is especially important for designs with a small number of produced parts, significant costs involved for a design change, or large prediction uncertainties.

Future research is in progress to improve the method described above. An estimation of the time required for a change and the resulting costs due to delay is also under progress.

Acknowledgements

This research was possible with the NSF grant # 970 2797 through the Division of Design, Manufacturing and Industrial Innovation and GE Plastics.

References

Devore, J. L. (1995). *Probability and Statistics for Engineering and the Sciences*, Wadsworth.

Papoulis, A. (1991). *Probability, Random Variables, and Stochastic Processes*, McGraw-Hill.

Roser, C., and Kazmer, D. (1998).“Robust Product Design for Injection Molding Parts.” *ANTEC*, New York.

Roser, C., and Kazmer, D. (1999).“Risk Effect Minimization using Flexible Design.” *Design Engineering Technical Conferences, Design for Manufacturing Conference*, Las Vegas, California.

Steward, D. V. (1981). “The Design Structure System: A Method for Managing the Design of Complex Systems.” *IEEE Transactions on Engineering Management*, EM-28(3), 71-74.

Suresh, K. (1997).“Monte Carlo Analysis Techniques Revisited.” *1997 ASME Design Engineering Technical Conferences*, Sacramento, California.

Thornton, A. C. (1999). “Optimism vs. Pessimism: Design decisions in the face of process capability uncertainty.” *accepted to: ASME Journal of Mechanical Design*.

Preprint of: Roser, Christoph, and David O Kazmer. “A Method for Robust Flexible Design.” In *Proceedings of the Annual Technical Meeting of the Society of Plastic Engineers*, 3:2948–52. Orlando, Florida, USA: Society of Plastics Engineers, 2000.

Tables

Design	Total Cost	Yield due to Uncertainty	Probability of Design Selection
D^R_1	C^T_1	P^U_1	P^U_1
D^R_2	C^T_2	P^U_2	$P^U_2 - P^U_1$
D^R_3	C^T_3	P^U_3	$P^U_3 - P^U_2$
…	…	…	…

Table 1: Design Change Options Overview

Design Variable	Nom.	Design Response	Nom.
Mold Temperature	T^{Mold}	Melt Pressure	P^{Melt}
Wall Thickness	W	Shrinkage	S
Number of Tools	N	Clamp Force	F^{Clamp}
Material Type	M	Production Time	t

Table 2: Design Variables and Design Responses

Nr.	C^U	P^N	P^C	P^U	Comment
1	$8.85	0.988	0.827	0.837	C^T Optimal
2	$30.94	1.000	0.982	0.982	P^C Optimal
3	$8.90	0.993	0.848	0.854	t increased

Table 3: Different Investigated Designs

j	C^U	P^N	P^C	P^U	C^T
1	8.91	0.993	0.848	0.854	8.91
9	8.95	0.997	0.874	0.877	8.95
2	8.98	0.999	0.900	0.901	8.99
10	9.05	0.999	0.918	0.918	9.05
5	10.12	1.000	0.971	0.971	11.74
13	10.25	1.000	0.975	0.975	11.86
7	30.91	1.000	0.980	0.980	36.52
15	30.94	1.000	0.982	0.982	36.55

Table 4: Design Flexibility Data

Preprint of: Roser, Christoph, and David O Kazmer. “A Method for Robust Flexible Design.” In *Proceedings of the Annual Technical Meeting of the Society of Plastic Engineers*, 3:2948–52. Orlando, Florida, USA: Society of Plastics Engineers, 2000.

Figures

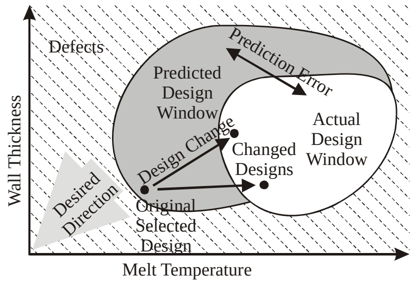

Figure 1: Flexible Design Incentive

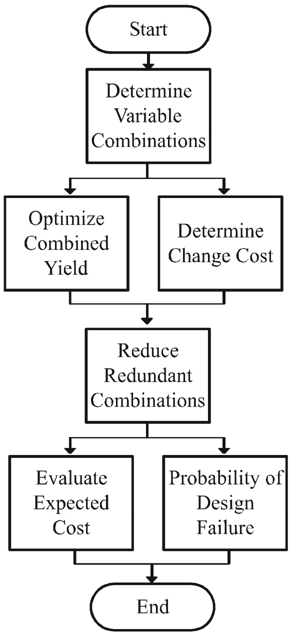

Figure 2: Methodology Overview

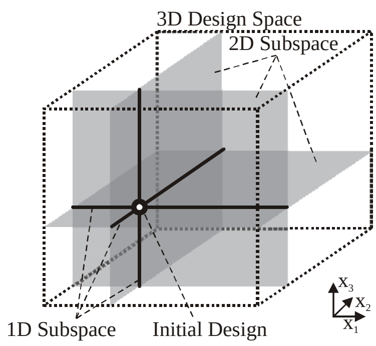

Figure 3: 3D Design Space and Sub-Spaces

Preprint of: Roser, Christoph, and David O Kazmer. “A Method for Robust Flexible Design.” In *Proceedings of the Annual Technical Meeting of the Society of Plastic Engineers*, 3:2948–52. Orlando, Florida, USA: Society of Plastics Engineers, 2000.

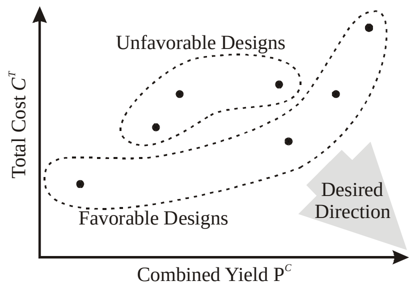

Figure 4: Design Flexibility

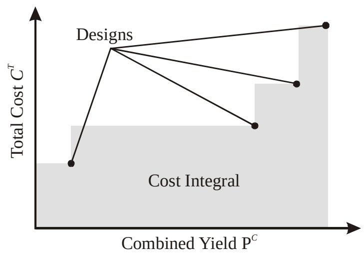

Figure 5: Expected Cost including Design Changes

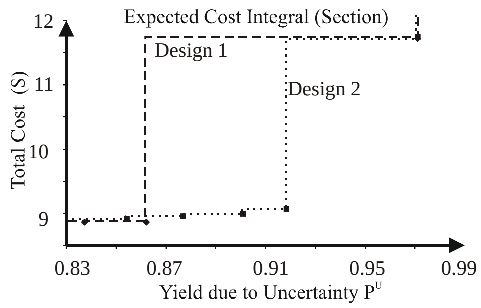

Figure 6: Flexibility Comparison

Keywords

Flexible Design, Robust Design Change, Design Failure, Uncertainty